Testing the Edwards hypothesis in spin systems under tapping dynamics

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Abstract. The Edwards hypothesis of ergodicity of blocked configurations for gently tapped granular materials is tested for abstract models of spin systems on random graphs and spin chains with kinetic constraints. The tapping dynamics is modeled by considering two distinct mechanisms of energy injection: *thermal* and *random* tapping. We find that ergodicity depends upon the tapping procedure (i.e. the way the blocked configurations are dynamically accessed): for thermal tapping ergodicity is a good approximation, while it fails to describe the asymptotic stationary state reached by the random tapping dynamics.

1. Introduction

The probabilistic description of dissipative dynamical systems is a central issue of modern statistical physics. In general, the non-equilibrium nature of the stationary state makes a general principle analogous to the Boltzmann ergodic hypothesis for Hamiltonian systems hard to find. The situation may be fortunate in the case of gently tapped granular materials, where the dynamics consists of cycles in which the system passes from a blocked configuration to another through discrete injection of mechanical energy (a tap) followed by a zero temperature relaxation under gravity. The observation of a reversible branch in the curve of the asymptotic packing density versus the tapping amplitude, suggests the existence of a stationary regime in which the packing density depends monotonically on the vibration intensity [1]. On the other hand, it is also known that macroscopic features of mechanically stable packings (e.g. the packing density) depends on their collective handling, i.e. the specific tapping procedure as borned out by simulations [2] and experiments [3]. In this situation one can ask for the invariant dynamical measure which describes the sampling of the blocked configurations, and how it depends on the energy injection mechanism.

‡ Until December 31, 2001.

The simplest hypothesis was made some times ago in a series of papers by Edwards and co-workers where the uniform distribution over the blocked states of given density was assumed independently of the tapping procedure, provided it is extensive [4, 5]. This proposal is particularly attractive as it leads by construction to a thermodynamic framework analogous to that of ordinary thermal systems. In particular, it leads to the concept of compactivity, which for granular matter would play the same role of the temperature in thermodynamic systems.

Effective temperatures also appear in the description of glassy systems undergoing aging dynamics, which are by their nature far away from their stationary state [6]. Their occurrence can be justified by supposing that, although ergodicity does not hold at the level of a single trajectory, trajectories corresponding to different initial conditions and thermal histories sample finite life-time states with asymptotically uniform measure [7]. The issue has been investigated numerically with positive answer in 3D Lennard-Jones glasses [8]. In the aging dynamics of a non-thermal kinetic lattice-gas like the Kob-Andersen model [9, 10], the generalised effective temperature [11], as well as more local observables as the structure function, appears to be in agreement with the corresponding ones computed from the Edwards measure on the blocked states [12]. Similar results have been obtained in a realistic model of granular media under shear [13]. Further hints in favour of the Edwards hypothesis have been presented in recent studies [14, 15, 16, 17, 18, 19] on various kinds of spin models with tapping dynamics. These results suggest a unified thermodynamic framework to describe aging glasses and gently tapped granular systems [20].

Despite the fascination and the strong predictive power of a statistical mechanical construction, it is at present not clear to what extent and generality it actually applies to granular and glassy systems, and what would they be the underlying reasons. It is quite natural in that context to look at the problem in abstract models, which while mimicking the tapping energy injection mechanism and subsequent dissipation, are easily amenable to numerical and analytical investigations thus allowing to test the hypothesis in a fine detail. Here we study two models of one dimensional kinetically constrained systems (section 2), and some spin models on diluted random graphs (section 3), by considering two distinct energy injection mechanisms, which we call *thermal* and *random* tapping. We find the uniform measure to be a good approximation for the thermal tapping, with improving accuracy at decreasing tapping intensities. While for the random tapping we observe systematic deviations from the uniform measure for all finite tapping intensities. Moreover, in the case of kinetically constrained systems, the validity of the approximation for thermal tapping dynamics does not warrant its extension to the aging regime. In the case of random graph models we find that the validity of the uniform measure also depends on whether or not there are neutral moves (single spin flips which do not change the energy) and propose a modified measure for the former case.

2. Kinetically constrained spin chains

The first two models we consider are a variant of the facilitated Ising spin chains first introduced by Fredrickson and Andersen [21, 22], and its asymmetric version introduced by Jäckle and Eisinger [23]. These are abstract toy models of glassy behaviour whose Hamiltonian is simply

$$E = -\sum_{i=1}^{N} n_i \,, \tag{1}$$

where the $n_i = 0, 1$ are binary variables and the index i runs over the sites of a chain of length N with periodic boundary condition. Their dynamics is defined by the following kinetic constraints:

- **Symmetric model (model S)** A variable can flip with a non-zero rate only if at least one of its neighbouring variable is equal to zero. Specifically, the variables are randomly updated according to the transition matrix

$$\mathcal{W}(n_i \to 1 - n_i) = (1 - n_{i+1}\, n_{i-1}) \min\left[1, \exp(-\Delta E/T)\right] . \tag{2}$$

- **Asymmetric model (model A)** A variable can flip with a non-zero rate only if its *left* neighbouring variable is equal to zero. In this case the transition matrix is

$$\mathcal{W}(n_i \to 1 - n_i) = (1 - n_{i-1}) \min\left[1, \exp(-\Delta E/T)\right] . \tag{3}$$

With these rules detailed balance is satisfied and the Markov chain associated with the dynamic evolution at non-zero temperature is irreducible on the full configuration space with the exception of the configuration with the lowest energy (all the spins equal to one so the kinetic constraints prohibit a dynamical evolution). Therefore the approach to the canonic equilibrium distribution is guaranteed. However, after a quench at low temperature the density of zeros become smaller and smaller and hence the relaxation become sluggish as the kinetic constraints are hardly satisfied. In spite of their simple equilibrium measure, the finite temperature dynamics of these models does not seem to be exactly solvable, but several important results are known [23, 24, 25, 26, 27, 28]. In particular, the characteristic equilibration time at low-temperature diverges as $\tau \sim \mathrm{e}^{b/T}$ for the model S [24, 25]; and with a super-Arrhenius law $\tau \sim \mathrm{e}^{a/T^2}$, for the model A [23, 26, 27].

One reason of special interest in these models is that they provide a more severe test of the validity of the Edwards hypothesis since they are characterised by the same entropy of blocked configurations – though their relaxational dynamics is qualitatively different§. Moreover, they offer the advantage that the Edwards measure can be exactly computed and the analytic results compared with the corresponding ones obtained from numerical simulations of tapping. In the following we will be mainly interested in the stationary state reached by these systems when they are submitted to a periodic non-relaxational perturbation that mimics two different extensive tapping mechanisms.

§ There is actually a continuous class of dynamical models sharing the same entropy of blocked configurations, see [28].

2.1. Tapping dynamics

The tapping is modelled by cycles consisting in two dynamical steps: an "energy injection" step (called a tap for short) followed by a zero temperature relaxation until blocking occurs [2, 29, 30, 14, 15, 16]. During the tapping the spins are randomly updated according two distinct ways:

(i) **Thermal tapping (T)** The system undergoes a Monte-Carlo sweep at temperature T with transition matrix specified by Eq. (2) or (3) depending on the model.

(ii) **Random tapping (R)** Each variables is flipped in parallel with probability $p \in (0, \frac{1}{2}]$, irrespective of the kinetic constraints.

During the tapping dynamics the detailed balance is broken, and after a long enough time the system is expected to reach a steady state regime in which the energy injected into it is in average equal to that dissipated in the zero temperature relaxation steps. We also checked that the steady state is independent of the initial configuration (with the exception of the lowest energy configuration for the thermal tapping). Both tapping mechanisms coincide in the "infinite tapping limit" where a tap consists in reinitialising completely the system in a random configuration. The dynamics in this limit has been recently solved by De Smedt et al. [31] in kinetic 1D models similar to the ones we study here, finding results in full agreement with ours. For an analytical approach to the thermal tapping dynamic of the 1D Fredrickson-Andersen model see also ref. [14]. Note that the blocking condition, namely that zeros are isolated, is obviously independent of the tapping mechanism. However, the statistical properties of the blocked configurations in the stationary state might be (and actually are, as we will see) dependent on the way they are typically accessed. The set of blocked configurations for the model S and A is the same, and one can compute their number $\mathcal{N}(e)$ as a function of the energy density e, through simple combinatorial arguments [28]. In the thermodynamic limit this number is exponentially large, and the Edwards entropy, which is by definition $s(e) = \frac{1}{N} \log \mathcal{N}(e)$, reads:

$$s(e) = e \log \frac{1+2e}{e} + (1+e) \log \frac{-1-2e}{1+e}\,, \tag{4}$$

from which one gets the inverse temperature or "compactivity"

$$\beta(e) \equiv \frac{\partial s}{\partial e} = \log \frac{(1+2e)^2}{-e(1+e)}\,. \tag{5}$$

We have performed extensive numerical simulations of both models A and S with both thermal and random tapping dynamics. We used spin chains of length $N = 2^{10}$, 2^{15}, checking finite-size effects against $N = 2^{20}$. The observable computed in the steady state regime were typically averaged over samples of size 10^6, 10^7. In Fig. 1 we plot the energy density in the stationary state vs the tapping amplitude for the four possible cases we have examined. We remark that curves corresponding to the two energy injection mechanisms are rather different. The random tapping explores only configurations within quite a narrow interval of energy, and the zero tapping limit of the steady state

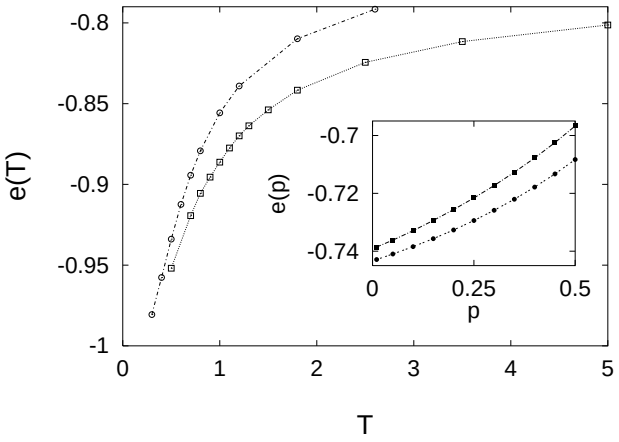

Figure 1. Energy density vs tapping amplitude in the stationary state of the tapping dynamics of the model S (circle) and model A (square). Open symbols represents the *thermal* tapping (main figure), while solid symbols correspond to the *random* tapping (inset).

energy seems to converge to a value higher than the ground state, where the blocked state entropy is still extensive (see the inset of Fig. 1). Decreasing p below the value 0.1 does not yield substantially lower energies but only makes longer the relaxation time to the stationary state. In the weak tapping regime we have explored, $10^{-1} \geq p \geq 10^{-5}$, this relaxation time goes like $\tau_{\rm rel} \sim p^{-1}$. With the thermal tapping mechanism instead, both models A and S are able to explore a wider energy range and they appear to reach the ground state as the tapping amplitude decreases to zero. Blocked configurations reached with random tapping are therefore less compact of those reached with thermal tapping, leading for these models to a non-universal (i.e. dependent on the dynamical mechanism) asymptotic packing density. Also notice that with random tapping the asymptotic energy density of the model A at a given p is lower than the corresponding one for the model S. This is easily understood as the asymmetric constraint is stronger than the symmetric one and hence the probability of the transition $0 \to 1$ in the zero temperature relaxation step is higher in the model S than A. For the thermal tapping instead just the opposite happens. In this case during the energy injection step the spins can only be flipped by respecting the kinetic constraints. This gives a lower number of spin-flip transitions in the model A with the respect to S (as the latter is characterised by a weaker constraint), which eventually results in a lower asymptotic energy for the model A.

The question that naturally arises in this context is whether models with different energy vs tapping amplitude plots but with the same set of blocked configurations may also share the same tapping thermodynamics. In order to investigate this point we measure several observables in the stationary state of tapping dynamics and compare their value with the corresponding observable analytically computed with the Edwards measure.

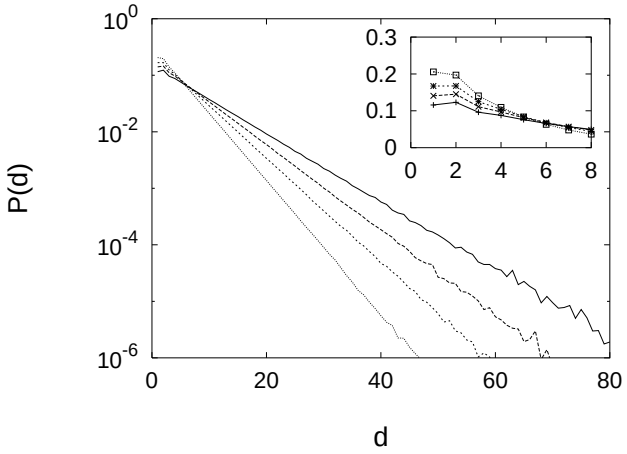

Figure 2. Probability distribution of domain length, $P(d)$, in the stationary state of the *thermal* tapping dynamics of model A. Tapping temperature $T = 1.0$, 1.3, 1.8, and 3.5. The distribution is exponential for long domain size, but deviations (stronger the higher the tapping amplitude) from the pure exponential can be detected for short domains (see inset). Similar results were obtained for the *random* tapping and for the Ising chain with symmetric constraint (model S).

We first examine the probability distribution of domains size. A domain of size d is defined here as a sequence of d ones enclosed by zeros. The explicit computation reveals that within the uniform measure, the distribution is exponential:

$$P(d) = \frac{1+e}{-e}\left(\frac{2e+1}{e}\right)^{d-1}, \quad d \geq 1\,. \tag{6}$$

Notice that this exponential distribution corresponds to independently "throwing" the number of zeros in a given interval, compatibly with the blocking conditions and with no further correlations. Any deviation from the exponential on the contrary implies correlations induced by the dynamics, and our test can be seen as a measure of these correlations. In Fig. 2 we show the function $P(d)$ as obtained from the thermal tapping of the A model. We see that while the exponential distributions works excellently for long domain size, deviations are detected for short domains as a consequence of the short-range correlations created by the kinetic constraints. Similar results are obtained for the model S and in both models, more pronounced deviations (not shown) are found for random tapping dynamics. In order to get a quantitative estimate of deviations we measure the mean-squared fluctuations of domain length σ_d^2, which in our case is given by

$$\sigma_d^2 = \frac{e(1+2e)}{1+e}\,. \tag{7}$$

Notice that the average domain length $\overline{d}$,

$$\overline{d} = \frac{-e}{1+e}\,, \tag{8}$$

is not a good observable since it is only determined by the blocking condition as a function of the energy, whatever the domain length probability distribution. In Fig. 3 we

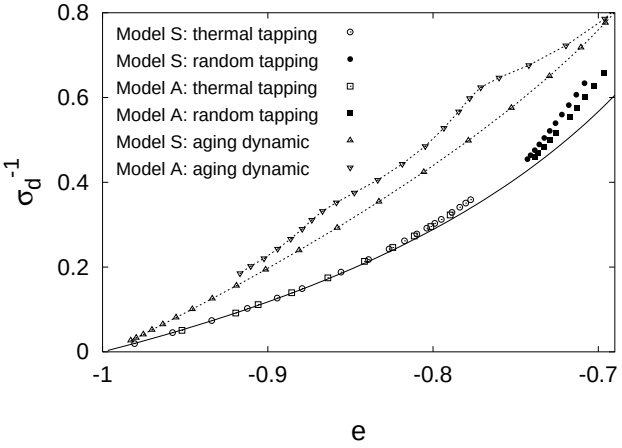

Figure 3. Inverse of the mean squared fluctuation length σ_d^{-1}, vs energy density e. Circle (model S) and square (model A) represent numerical results for the stationary state of the thermal (open symbols) and random (solid symbols) tapping dynamics in the constrained Ising chains. The full line represents the analytical result obtained from Eq. (7). For comparison also shown are the numerical results for the non-stationary relaxational dynamics after a quench at temperature $T = 0.2$ (model S, upward triangle) and $T = 0.25$ (model A, downward triangle).

show σ_d^{-1} vs e for both models and both tapping mechanisms. We find that the flatness assumption over blocked configurations with fixed energy works well for thermal tapping at low energy and does not depend on the nature of kinetic constraints; while small but systematic deviations are found at increasing energy. For the random tapping instead the deviations are found to be quite large at any tapping intensity, showing that the sampling of configurations is not ergodic with this energy injection mechanism. One can easily check that the energy interval explored with the random tapping dynamics corresponds to a small region around the maximum of the entropy of blocked configurations Eq. (4), where $e(s_{\max}) \simeq -0.7236$. In this region the compactivity Eq. (5), is very small and the Edwards hypothesis is not expected to hold [4, 12].

For comparison we have also studied the aging dynamics, i.e. thermal relaxation at a low temperature T starting from a high energy random configuration. Given the simple one-dimensional nature of the model, the system eventually equilibrates to the canonical distribution. However, before equilibrium is reached, the system enters a scaling regime during which the average domain length grows as $\overline{d}(t) \sim t^{aT}$ for the model A [27], and in a purely diffusive way, $\overline{d}(t) \sim t^{1/2}$ [26, 28], in the model S. In this regime the domain sizes probability distribution is not exponential, and although the average domain length closely approaches Eq. (8) (see the second of refs. [28]), the inverse of the mean squared fluctuation length remains far from the Edwards value in the whole scaling regime for both models, except at exceedingly low energy (see, Fig. 3).

Another aspect of the Edwards thermodynamic construction concerns the behaviour of the energy fluctuations. By standard thermodynamics, in the regimes well approximated by the Edwards hypothesis, the *spatial* fluctuations of the energy σ_e^2,

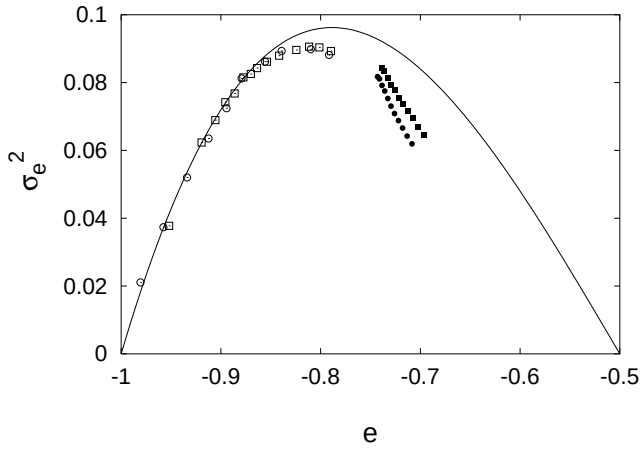

Figure 4. Test of the fluctuation-dissipation relation Eq. (9), in the stationary state of the tapping dynamics of Ising chains with kinetic constraints. Symbols correspond to the mean-squared energy fluctuations σ_e^2 vs energy density e, numerically obtained with the *thermal* (open symbol) and the *random* (solid symbol) tapping dynamics for the model S (circle) and model A (square). The full line represents the analytic result Eq. (10).

should follow the canonical relation

$$\sigma_e^2 = -\frac{\partial e}{\partial \beta}\,, \tag{9}$$

which in our case gives

$$\sigma_e^2 = e(1+e)(1+2e)\,. \tag{10}$$

One may wonder whether in that regime the *temporal* fluctuations follow the same law, hinting for a canonical distribution of the blocked states. We find that the energy fluctuations in the stationary state of the tapping dynamics are essentially Gaussian distributed. In Fig. 4 we compare the mean-squared fluctuations of energy σ_e^2, with the analytic result. We see that in the region where the uniform hypothesis works well, the fluctuations follow within numerical error the law implied by the canonical statistics, with improving accuracy as the tapping intensity is decreased. Consistently with our previous results we find that for random tapping the Eq. (10) is violated.

Small but systematic deviations at higher tapping intensity can also be observed in the random tapping dynamics of a one dimensional Ising chain recently studied by Dean and Lefèvre [17]. Interestingly, we have found that the *thermal* tapping dynamics of this model gives results similar to the random tapping. Hence, at variance with the kinetically constrained Ising chains, in the Dean-Lefèvre model there is substantially no difference between the two distinct energy injection mechanisms.

In conclusion, our results show that for thermal tapping the Edwards measure is a good approximation independently of the kinetic constrains, but it cannot be considered exact as systematic deviations appear at increasing tapping intensity. Random tapping instead prevents the system from reaching high compactivity states. This results in a

disagreement with Edwards measure, showing that the flatness assumption can be very sensitive to the nature of the energy injection mechanism and its interplay with blocking condition.

3. The three-spin model on random graphs

In this section we focus on the results of random and thermal tapping applied to three-spin models defined on a random graph with fixed connectivity k. The model is defined by the Hamiltonian

$$H = -\sum_{l<m<n} C_{lmn} S_l S_m S_n \tag{11}$$

where the connectivities C_{lmn} are invariant under permutation of the indices and are chosen at random with the constraint that $\sum_{m<n} C_{lmn} = k \, \forall l$. The three-spin model (albeit with fluctuating connectivities) has been used to model granular compaction [15], since it features states with locally minimal energies (satisfied plaquettes are +++,–+, and permutations of the latter), which however may be globally incompatible with one another. Under the term 'geometric frustration' the same mechanism is thought to be at the heart of the slow compaction of granulars. The model is also attractive as the aging evolution should obey mean field theory, where the asymptotic validity of flat measure on Thouless-Anderson-Palmer states is well established, and one can test whether tapping and glassy relaxational dynamics are by some means related [**?**]. However the fact that tapping is non-thermal complicates the issue - for instance for large tapping amplitudes one cannot expect a flat measure over blocked states to hold. Consider as an extreme case random tapping with $p = 1/2$, which corresponds to a series of quenches from random initial conditions. Since the typical blocked configurations do not - in general - have the largest basin of attraction among each other, we would not expect the flat measure over the blocked states to hold.

Configurations are deemed blocked if $h_l s_l \geq 0$ where $h_l = \sum_{m<n} C_{lmn} s_m s_n$ is the local field acting on each site. In order to check the Edwards hypothesis for such models, the statistical mechanics of blocked configurations of these models must be worked out. In principle, this would involve averaging the logarithm of the number of blocked configurations over the disorder, i.e. the different graphs (quenched average). From an analytical point of view, fixed connectivity graphs provide a simple testing-ground, since using the methods of [33, 34] one finds that for sufficiently high energies the annealed average of the number of blocked configurations gives the same result as the quenched average.

The number of blocked configurations $\mathcal{N}(e)$ at a given energy density e may be written easily as

$$\mathcal{N}(e) = \prod_l \left[\sum_{s_l=\pm 1} \sum_{h_l=-\infty}^{\infty} \delta\left(h_l - \frac{1}{2} \sum_{m,n} C_{lmn} s_m s_n \right) \right.$$
$$\left. \times \Theta\left(h_l s_l\right) \right] \delta\left(e - \frac{1}{3N} \sum_l h_l s_l \right) \ , \tag{12}$$

where $\delta(x)$, denotes a Kronecker-delta $\Theta(x)$ denotes a Heaviside step-function with $\Theta(x) = 1$ if $x \geq 0$ and 0 otherwise, and e^x denotes the standard exponential function. After using integral representations for the Kronecker-deltas and standard manipulations [35], one obtains the entropy of blocked states in the annealed approximation

$$s(e) = \frac{1}{N} \ln \langle\langle \mathcal{N}(e) \rangle\rangle = \mathrm{extr}_{a,b,\beta} \left[\beta e - \frac{8}{3} \left(a^3 + b^3\right) + \frac{2}{3} k (1 - \ln k) + \ln \left((2ab)^k \sum_{h=0(1)}^{\prime k} (\mathrm{e}^{\beta/3} \frac{a}{b})^h \left(\binom{k}{\frac{k-h}{2}} + \binom{k}{\frac{k+h}{2}} \right) \right) \right] \tag{13}$$

where the angular brackets denote the average over graphs of fixed connectivity k and a, b, β are to be determined by extremising this expression with respect to these three parameters. The sum over h proceeds in steps of two, as for even k thus only even local fields are possible, and likewise for odd values of k.

Having solved the self-consistent equations for the three parameters, one may also determine the fraction of sites g_h with a given value of the local field h (even h for even k and odd h for odd k)

$$g_h = \frac{(\mathrm{e}^{\beta/3} a/b)^{|h|} \binom{k}{\frac{k-h}{2}} \left[1 + \delta(h)\right]}{\sum_{h=0(1)}^{\prime k} (\mathrm{e}^{\beta/3} a/b)^h \left[\binom{k}{\frac{k-h}{2}} + \binom{k}{\frac{k+h}{2}} \right]} \ . \tag{14}$$

For $k > 3$ the fraction of sites with a certain local field serves as a convenient test of the Edwards hypothesis by comparing the values of g_h reached asymptotically with those predicted by the flat measure over all blocked configurations at the asymptotic energy. (For $k = 3$ and for a symmetric distribution of the local fields, the fraction of sites with $h = 1$ versus those of $h = 3$ is a unique function of the energy in all configurations).

In the following we compare the results of thermal and of random tapping for $k = 5$ and $k = 6$. We use system sizes of $N = 10^4$. An asymptotic state was typically reached after 10^6 taps in all but the lowest intensities, where up to 3×10^6 taps were necessary. Since graphs of fixed connectivity are highly homogeneous no sample averaging was necessary. In Figs. 5 and 6 the asymptotic results averaged over 1000 steps with the errorbars giving the standard deviation of the energy and the fraction of sites with a given local field also measured over 1000 steps. We plot the fraction of sites of a given local field against the asymptotic energy and compare the results with the prediction of (14).

As expected there are discrepancies between the numerical results of the tapping dynamics and the analytical results for the flat measure at high amplitudes of both thermal and random tapping. However also at low amplitudes small discrepancies are found. These are probably due to a dynamical slowing down and diverging equilibration times at low temperatures.

The situation is however more drastic in the case of $k = 6$, shown in Figs. 7 and 8 where both for thermal and for random tapping respectively the flat measure does not agree with the numerical results at any tapping amplitude.

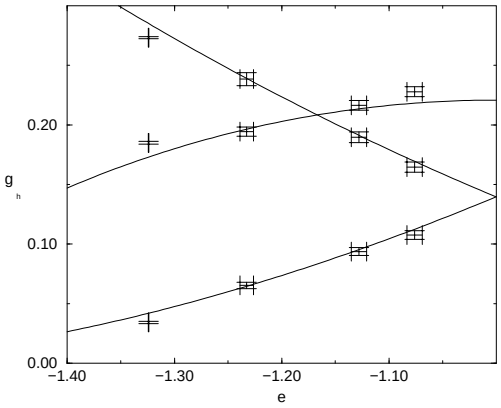

Figure 5. Asymptotic results for connectivity $k = 5$ and *thermal* tapping with $T = .5, 2, 2.86, 5$ (from left to right). The solid lines give the corresponding fractions of sites with $h_i = 1, 3, 5$ according to the flat measure (bottom to top on the lhs).

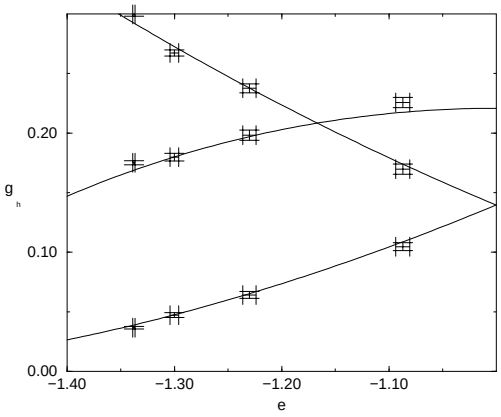

Figure 6. Asymptotic results for connectivity $k = 5$ and *random* tapping with $pN = 300, 500, 1000, 2000$ (lower tapping amplitudes did not yield substantially lower energies). The solid lines give the corresponding fractions of sites with $h_i = 1, 3, 5$ according to the flat measure (bottom to top on the lhs).

This effect is due to a simple but crucial difference between graphs of odd and even constant connectivity: Graphs with even connectivity qualitatively differ from those with odd connectivity since in the former case sites may have their local magnetic field equal to zero so their spins may be free to flip without changing the energy. The dynamics of these spins is crucial at low temperatures as they correspond to neutral directions in phase space [36].

In odd-connectivity graphs such neutral directions are absent, whereas in generic graphs with fluctuating connectivity as considered in [15] they are also present.

The spins with zero magnetic field may be thought of as being exposed to a continuous tapping process even during the quench phase. The idea that this process

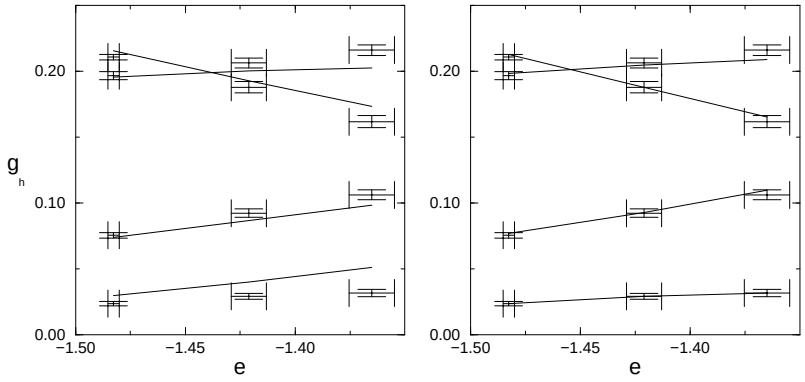

Figure 7. Asymptotic results for connectivity $k = 6$ and *thermal* tapping with $T = 2, 2.86, 5$ (from left to right, lower amplitudes did not yield substantially lower energies), plotting the fractions of sites with $h_i = 0, 2, 4, 6$ (bottom to top on the lhs) against the energy. In the left graph, the solid lines give the corresponding analytic result according to the uniform measure. In the graph on the right hand side, the same numerical results are plotted and compared to the restricted measure plotted as solid lines.

distorts the flat measure may be used to promote the fraction of spins with zero local field to a relevant macroscopic variable, like the energy. In fact, a better comparison with the dynamics is obtained considering the ensemble of all blocked configurations of a given energy *and* of a given fraction g_0 of sites with zero local field. The calculation is a simple variation on (13) and yields

$$g_h = \frac{(\mathrm{e}^{\beta/3}a/b)^{|h|}\binom{k}{\frac{k-h}{2}}\left[1+\delta(h)\right]\mathrm{e}^{-\hat{g}_0\delta_{h,0}}}{\sum_{h=0(1)}^{\prime k}\mathrm{e}^{-\hat{g}_0\delta_{h,0}}\left(\mathrm{e}^{\beta/3}a/b\right)^h\left[\binom{k}{\frac{k-h}{2}}+\binom{k}{\frac{k+h}{2}}\right]} \ . \tag{15}$$

where the order parameters are determined by the extremal condition in

$$s(e) = \mathrm{extr}_{a,b,\hat{g}_0,\beta}\left[\hat{g}_0 g_0 + \beta e - \frac{8}{3}\left(a^3+b^3\right) + \frac{2}{3}k(1-\ln k) + \right.$$
$$\left. \ln\left((2ab)^k \sum_{h=0(1)}^{\prime k}\mathrm{e}^{-\hat{g}_0\delta_{h,0}}\left(\mathrm{e}^{\beta/3}\frac{a}{b}\right)^h \times \left(\binom{k}{\frac{k-h}{2}}+\binom{k}{\frac{k+h}{2}}\right)\right)\right] \ . \tag{16}$$

To test this new ensemble we compare the values of g_2, g_4, g_6 at the asymptotic state with those given analytically by the restricted ensemble at the asymptotic values both of the energy and of g_0. The results are shown in Figs. 7 and 8. Except at high amplitudes, the numerical and analytical results agree very well. Clearly more information on the

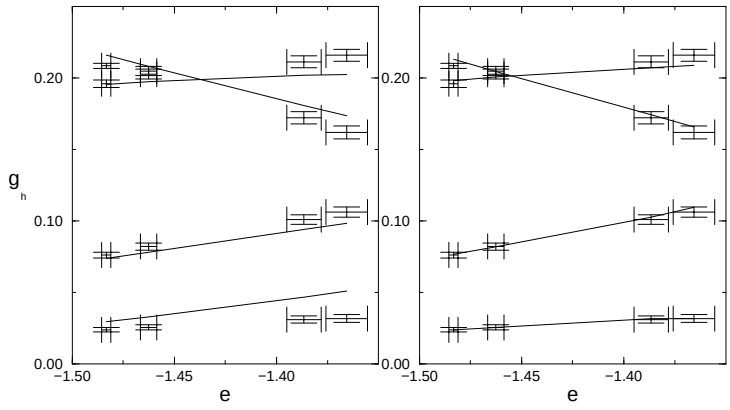

Figure 8. Asymptotic results for connectivity $k = 6$ and *random* tapping with $pN = 300, 500, 1000, 2000$ (lower tapping amplitudes did not yield substantially lower energies), again plotting the fractions of sites with $h_i = 0, 2, 4, 6$ (bottom to top on the lhs) against the energy. In the left graph, the solid lines give the corresponding analytic result according to the uniform measure. In the graph on the right hand side, the same results numerical results are plotted and compared to the restricted measure plotted as solid lines.

blocked states is used in the restricted measure, so some improvement of the fit between analytical and numerical results is expected solely on these grounds. Similarly the agreement in the case of the fraction of sites with zero local field g_0 is solely due to the fitting. Nevertheless the fact that the flat measure fails in the case of even-connectivity graphs shows the crucial role of sites with zero magnetic field. The restricted measure is the simplest way of modifying the flat measure in this case.

Of course the relevance of such neutral moves to realistic models of e.g. of granular is debatable, however loosely constrained particles termed 'rattlers' have been found to cause subtle dynamical effects in simulations of granular particles [37].

4. Summary and Conclusions

In this paper we have studied stationary dynamical measures of several abstract spin models endowed with two kinds of energy injection mechanism: random and thermal tapping. We first considered two kinetically constrained Ising chains (with symmetric and asymmetric constraints) having the same entropy of blocked states. We find that, in the case of the thermal tapping, the Edwards measure gives a good approximation for the observables we studied, *independently* of the kinetic constraints. This can be understood as the uniform measure implies uncorrelated domains of up spins, and the dynamics does

not create spatial long range correlations. Despite that, small correlations are always dynamically induced and systematic deviations are found. As one could expect, the quality of the approximation improves as one goes towards lower tapping amplitudes and lower energies. Deviations are particularly evident in the case of random tapping showing that the energy injection mechanism may have a strong influence on the nature of the asymptotic stationary regime. We interpret these deviations as essentially due to the vanishing compactivity of blocked configurations reached by random tapping. We finally observed that non-ergodic sampling occurs during the aging as the purely relaxational dynamics of these Ising chains is a domain growth process. Nevertheless, this does not prevent the Edwards measure to be a good approximation for the steady state of the thermal tapping. This suggests that ergodicity in the stationary regime generally requires less stringent conditions than the aging dynamics.

Similar results are obtained in the case of the 3-spin model on the random hypergraph, where the Edwards measure gives a reasonable approximation both for thermal and for random tapping provided there are no neutral directions in phase space. In the latter case, realized by sites with zero local field, we introduced a restricted measure of blocked configurations with a *given* fraction of sites with zero local fields.

It is an open problem if there are systems where the uniform measure on blocked states is exact for tapping dynamics. In this paper we showed that at least for thermal tapping, where the energy injection step is correlated with the energy landscape, the Edwards hypothesis is a good approximation whose quality increases with decreasing tapping amplitude.

Acknowledgements

S.F. acknowledges interesting discussions with A. Barrat and J.M. Luck. We thank G. De Smedt, C. Godreche and J.M. Luck for communicating us their results before publication.

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